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Theoretical calculation and experimental study on the load distribution coefficient (LDC) of three-ring gear reducer

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Abstract: In this paper, primary manufacturing and assembling errors of three-ring gear reducer (TRGR) are analyzed. TRGR is a new transmission type whose eccentric phase difference between middle ring plate and side ring plates is 120°. Its mass of middle ring plate is equal to that of side ring plate or 180°, and its mass of middle ring plate is twice of that of side ring plate, which affects load distribution between ring plates. The primary manufacturing and assembling errors include eccentric error of eccentric sheath E_m, internal gear plate E_r and output external gear E_n. A new theoretical method is presented in this paper, which converts load on ring plates into the dedendum bending stress of ring plate to calculate load distribution coefficient (LDC), by means of gap element method (GEM), one of finite element method (FEM). The theoretical calculation and experimental study, which measures ring plate dedendum bending stress by means of sticking strain gauges on the dedendum of middle ring plate internal gear and side ring plate internal gears, are presented. The theoretical calculation and comparison with experiment result of LDC are implemented on two kinds of three-ring gear reducers whose eccentric phase difference between eccentric sheaths is 120° and 180° respectively. The research indicates that the result of theoretical calculation is consistent with that of experimental study. That is to say, the theoretical calculation method is feasible.

Key words: three-ring gear reducer (TRGR); load distribution coefficient (LDC); finite element method (FEM); gap element method (GEM)
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Three-ring gear reducer (TRGR) is one of transmission inventions in China. As a transmission device, it works on basis of few teeth difference transmission principle. Three-ring transmission structure is as shown in Fig. 1. Three-ring transmission consists of two high-speed input shafts 1 with three eccentric shaft sheaths respectively whose phase difference is 120° or 180°, one low-speed output shaft 2, three ring plates 3 with internal gear and external gear 4 assembled on output shaft. All shafts are parallel. In this paper, the first kind of TRGR[1] whose eccentric phase difference between eccentric sheaths equals 120° is mentioned and the second kind of TRGR[2] whose eccentric phase difference between eccentric sheaths equals 180° is presented. Three ring plates are assembled with high-speed shafts 1. They mesh with external gear 4 on a plane. It has a big transmission ratio. The two input shafts might be a driver separately or drivers simultaneously.

The transmission ratio of three-ring gear reducer is shown as follows:

$$i = -\frac{Z_4}{Z_3 - Z_4} \qquad (1)$$

where Z_4 and Z_3 represent the teeth of external gear and internal gear of the TRGR.

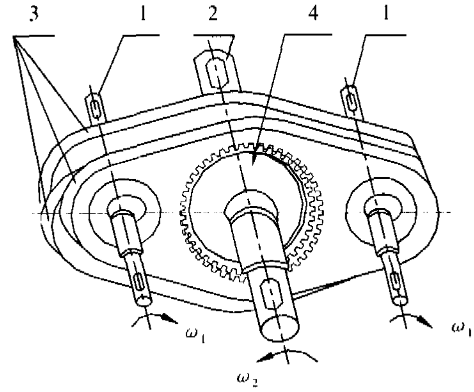

Fig. 1 The basic structure of three-ring transmission

Three-ring gear transmission has the following characteristics:

Their teeth profile curvatures are in the same direction, so it is small in relative normal curvature and big in load capacity.

The distance between gear teeth of internal gear plate and external gear is very small, so multiple teeth meshing always occur to increase its load capacity.

Because of its excellent load capacity and overload capacity, it has been used widely in numerous relevant fields, such as national defence, chemical engi-

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neering, light industry, weight-lifting and transportation, construction engineering, food industry and manufacturing engineering, etc.

1 Load Distribution and Error Analysis

From Fig. 1, we can see that each internal gear plate is a double-crank mechanism and three internal gear plates drive external gear on output shaft in the same time. Theoretically, load on each internal gear plate is equal at all time. But in fact, because of parts distortion and unavoidable manufacturing and assembling errors, load on each internal gear plate is not equal. That is to say, load distribution is not balanced between internal gear plates and this will affect on their load capacity.

The main factors that affect on load distribution of TRGR are three eccentric errors, E_m, E_r and E_w represent the eccentric errors of eccentric sheaths, internal gears and external gear respectively. According to geometry relation of meshing transmission, three eccentric errors can be converted into equivalent central error on output shaft y_m, y_r and y_w, their maximal value can be expressed as follows:

$$\begin{aligned} y_{m\max} &= E_m \\ y_{r\max} &= E_r/2\cos\alpha' \\ y_{w\max} &= E_w \end{aligned} \tag{2}$$

where, α' represents meshing angle.

Adding load equilibrating mechanism to compensate part error is an effective method to reduce and eliminate the effects of the three errors. The aim of the paper is to calculate theoretically LDC of two kinds of TRGR without load equilibrating mechanism by means of non-linear finite element method and confirm the result through experimental study. And this might provide necessary technical specification to design load equilibrating mechanism. Furthermore, load distribution capacity of two kinds of TRGR is compared and analyzed.

The equivalent radial displacement y caused by three eccentric errors on external gear shaft equals the vector sum of equivalent central errors. It may be expressed as follows

$$\boldsymbol{y} = \boldsymbol{y}_m + \boldsymbol{y}_r + \boldsymbol{y}_w \tag{3}$$

In fact, the equivalent central errors are cycle functions of time and their phase vectors conform to equiprobability density distribution. Their phases are impossible same at any time, so it is not right to calculate the equivalent central error through summing the maximal value of these three eccentric errors. It is reasonable to calculate the equivalent central error through square sum of these three eccentric errors. Referring to the probability calculation method of maximal floating value in Ref. [3], the maximal value of the equivalent radial displacement $y_{\max}$ is expressed as follows:

$$y_{\max} = \sqrt{E_m^2 + E_r^2/4\cos^2\alpha' + E_w^2} \tag{4}$$

According to our design parameter of prototype HITSH145 type TRGR, the three eccentric errors can be given as follows:

$$\begin{aligned} E_m &= 0.010\ \text{mm} \\ E_r &= 0.050\ \text{mm} \\ E_w &= 0.050\ \text{mm} \\ \alpha' &= 37.356° \end{aligned} \tag{5}$$

So the maximal value of the equivalent radial displacement $y_{\max} = 0.060$ mm. In other words, three eccentric errors can cause radial displacement of 0.060 mm on external gear center without load equilibrating mechanism on the reducer.

2 Calculation Method

The LDC in planetary transmission is an important standard to judge its load distribution capacity. In general, it can be defined as a ratio of the maximal load to the average load on planetary gears of a planetary transmission reducer. For TRGR, it can be defined as a ratio of the maximal load to the average load in ring plates. The LDC K_P can be expressed as follows:

$$K_p = \frac{\max(P_{1\max}, P_{2\max}, \cdots P_{n\max})}{\frac{1}{n_p}(P_1 + P_2 + \cdots + P_n)} \tag{6}$$

where, max represents to find the maximal value among $P_{1\max}$, $P_{2\max}$, $\cdots P_{n\max}$;

P_1, P_2, $\cdots P_n$ represent the average load on ring plates;

$P_{1\max}$, $P_{2\max}$, $\cdots P_{n\max}$ represent the maximal load on ring plates;

n_p represents the amount of ring plates.

Because the dedendum bending stress is in direct proportion to load on ring plates, the LDC K_P can be expressed as follows:

$$K_p = \frac{\max(\sigma_{1\max}, \sigma_{2\max}, \cdots \sigma_{n\max})}{\frac{1}{n_p}(\sigma_1 + \sigma_2 + \cdots + \sigma_n)} \tag{7}$$

where, max represents to find the maximal value among $\sigma_{1\max}$, $\sigma_{2\max}$, $\cdots$ $\sigma_{n\max}$;

σ_1, σ_2, $\cdots$ σ_n represent the average dedendum bending stress on ring plates;

$\sigma_{1\max}$, $\sigma_{2\max}$, $\cdots$ $\sigma_{n\max}$ represent the maximal dedendum bending stress on ring plates;

n_p represents the amount of ring plates.

Therefore, as long as the dedendum bending stress on ring plates can be obtained, the LDC K_P of this reducer can be obtained via expression (7).

Finite Element Method (FEM) is an effective numerical analysis method of structure design and mechanics analysis. Integrated Design Engineering Analysis Software (I - DEAS) is a practical integrated design and analysis software[4]. The few teeth difference reducer with n internal gear ring plates is shown in Fig. 2. Main stress on ring plates in Fig. 2 can be obtained through applying I - DEAS and Gap Element Method (GEM) on the condition that load and boundary constraints accord with actual working condition. So the LDC K_P of this few teeth difference reducer with n internal gear rings can be obtained via expression (7).

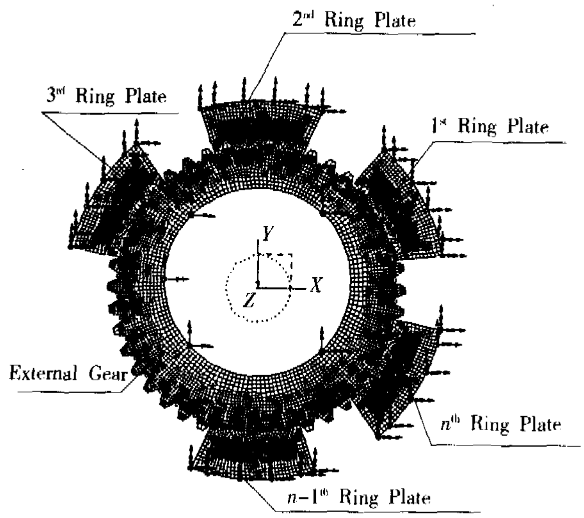

Fig. 2 Finite element analysis diagram of n ring plates few teeth difference reducer

3 Case Calculation

In this paper, the LDC of two kinds of TRGRs will be calculated. The first kind of TRGR is shown in Fig. 3 with three identical thickness of internal gear ring plates and phase difference is 120° among them. The transmission specifications are shown as follows:

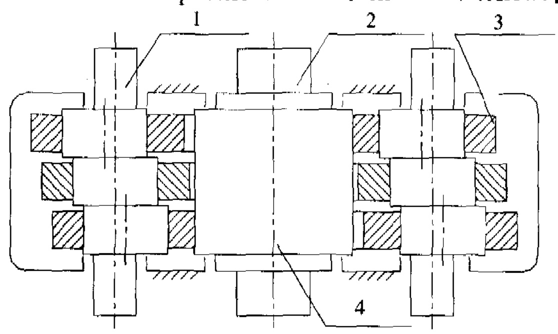

Fig. 3 The first kind of TRGR (Eccentric phase difference is 120°)

$z_1 = 42$, $z_2 = 44$, $m = 3.5$ mm, $h_a^* = 0.8$, $c^* = 0.3$, $x_1 = 1.14$ mm, $x_2 = 1.41$ mm, $b = 25$ mm, $d_{a1} = 159.3$ mm, $d_{f1} = 147.3$ mm, $d_{a2} = 157.7$ mm, $d_{f2} = 171.0$ mm.

According to three-ring transmission principle, we can learn that internal gears are driver ones and external gear are driven ones. So the boundary condition is to constrain the radial movement of internal gear pairs and the circumferential movement of external gear pair and the torque is added along the tangent of external gear. Gear meshing is within the range of plane stress issue. Therefore, we can calculate dedendum bending stress by applying four-node cell and its node thickness equal to the thickness of ring plates. According to the transmission structure, we set cell type, cell size and material property, and then generate 13 156 four-node cells and 13 852 nodes by applying Meshing Module. Its FEM analysis model is shown in Fig. 4. According to the above error analysis, constraint set and solution set can be established by applying I-DEAS and GEM. The calculation result by applying Solution Model is shown as follows:

$$\sigma_1 = 58.4 \text{ MPa}$$
$$\sigma_2 = 61.2 \text{ MPa}$$
$$\sigma_3 = 56.6 \text{ MPa}$$
$$\sigma_{\max} = 92 \text{ MPa}$$

So we can get that $K_p = 1.566$. Where, σ_i represents the average strain on each of ring plates.

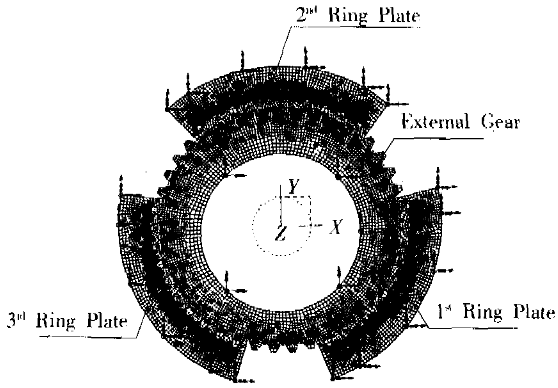

Fig. 4 Finite element analysis model of the first kind of TRGR

The second kind of TRGR is shown in Fig. 5 with three ring plates. Thickness of two side ring plates is identical and the middle one is twice as thick as side ones, and phase difference is 180° among them. In order to get over the dead point, both input shafts are driver ones. The transmission specifications are shown as follows:

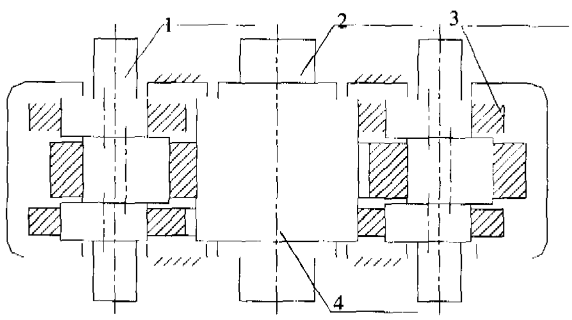

Fig. 5 The second kind of TRGR (Eccentric phase difference is 180°)

$z_1 = 42, z_2 = 44, m = 3.5$ mm, $h_a^* = 0.8, c^* = 0.3$, $x_1 = 1.14$ mm, $x_2 = 1.41$ mm, $d_{a1} = 159.3$ mm, $d_{f1} = 147.3$ mm, $b_1 = 38$ mm, $d_{a2} = 157.7$ mm, $d_{f2} = 171.0$ mm, $b_2 = 19$ mm.

In this paper, two internal gears on ring plates of the second kind of TRGR are machined synchronously by slotting machine. Two sheaths on ring plates are machined synchronously by precise lathe, boring holes and milling spline grooves and truncated into appropriate size. Thus we think they are identical and might join them into one which $b_2^* = 2b_2 = 38$ mm. According to plane stress issue, the boundary conditions and the torque are identical with the first kind of TRGR. Therefore, we can calculate dedendum bending stress by applying four-node cell and its node thickness equals to the thickness of ring plates. According to the transmission structure, we set cell type, cell size and material property, and then generate 10 503 four-node cells and 11 066 nodes by applying Meshing Module. Its FEM analysis model is shown in Fig. 6. According to the above error analysis, constraint set and solution set can be established by applying I - DEAS and GEM. The calculation result by applying Solution Model is shown as follows:

$$\sigma_1 = \sigma_3 = 56.4 \text{ MPa}$$

$$\sigma_2 = 52.5 \text{ MPa}$$

$$\sigma_{\max} = 73.2 \text{ MPa}$$

So we can get that $K_p = 1.344$.

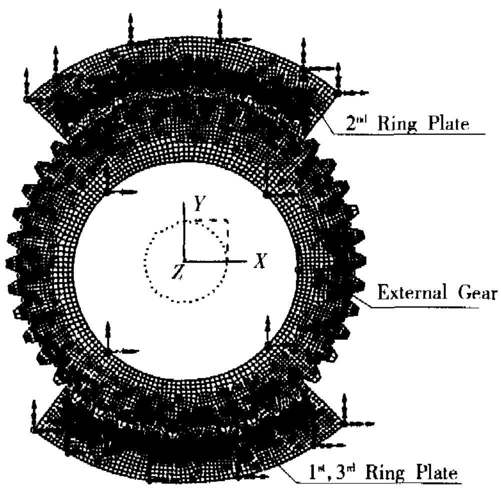

Fig. 6 Finite element analysis model of the second kind of TRGR

4 Experimental Study

Resistance strain gauges are applied as sensors to measure the load on internal gear ring plates in this experiment. Strain gauges are stuck on the same position of the dedendum of three ring plates. Their sticking position is shown as Fig. 7. Semi-bridge circuit is set up by Resistance strain gauges. Load signal is inputted into HY - 1232 A/D converter via dynamic resistance strain apparatus. Thus strain signals of ring plates are recorded. In this experiment, the amount of ring plate $n_p = 3$, the expression (6) can be simplified as follows:

$$K_p = \frac{3\max(P_{1\max}, P_{2\max}, P_{3\max})}{(P_1 + P_2 + P_3)} = \frac{3 \times \max(k_1\varepsilon_{1\max}, k_2\varepsilon_{2\max}, k_3\varepsilon_{3\max})}{(k_1\varepsilon_1 + k_2\varepsilon_2 + k_3\varepsilon_3)} \tag{8}$$

where, max represent to find the maximal value among $K_1\varepsilon_{1\max}$, $K_2\varepsilon_{2\max}$, $K_3\varepsilon_{3\max}$;

K_1, K_2, K_3 represent the given proportional coefficient;

ε_1, ε_2, ε_3 represent the average value of strain signal on ring plates;

$\varepsilon_{1\max}$, $\varepsilon_{2\max}$, $\varepsilon_{3\max}$ represent the maximal value of strain signal on ring plates.

The strain gauges made by Japanese KYOWA Company are used in this experiment. Their type No, is KFG - 02 - 120 - C1 - 11, gauge length is 0.2mm and its resistance is 119.8 ±0.2 Ω[5]. K_1, K_2 and K_3 differ very slightly from each other. So Eq. (8) can be simplified as follows:

$$K_p = \frac{3 \times \max(\varepsilon_{1\max}, \varepsilon_{2\max}, \varepsilon_{3\max})}{(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)} \tag{9}$$

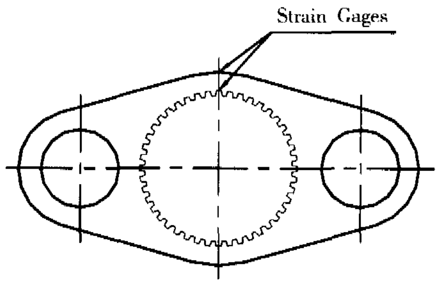

Fig. 7 Strain gage sticking position diagram

The maximal and average strain signal of three ring plates can be obtained through the experiment under the condition of different input speed and output torque. Then LDC can be calculated by means of expression (9). Experimental result diagram of LDC versus torque of the first kind of TRGR is shown in Fig. 8 and LDC versus torque of the second kind of TRGR is shown in Fig. 9 on condition that given input speed $n = 800$ r/min. the LDC of TRGR is define as the maximal value among the LDCs of three ring plates.

For the first kind of TRGR, the experiment results from Ref. [6] are as follows:

$$\varepsilon_1 = 46.86 \ \mu\varepsilon$$

$$\varepsilon_2 = 56.72 \ \mu\varepsilon$$

$$\varepsilon_3 = 28.20 \ \mu\varepsilon$$

$$\varepsilon_{\max} = 74.68 \ \mu\varepsilon$$

So we can get that $K_p = 1.70$.

For the second kind of TRGR, the experiment results are as follows:

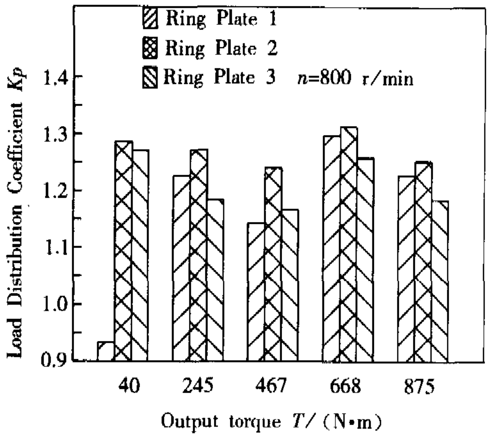

Fig. 8 Experimental diagram of LDC versus torque of the first kind of TRGR

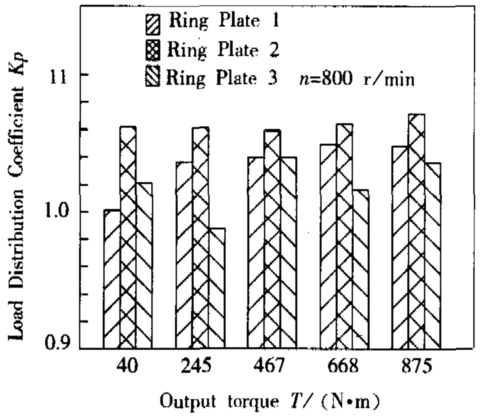

Fig. 9 Experimental diagram of LDC versus torque of the second kind of TRGR

$$\varepsilon_1 = 45.75\ \mu\varepsilon$$
$$\varepsilon_2 = 44.12\ \mu\varepsilon$$
$$\varepsilon_3 = 47.39\ \mu\varepsilon$$
$$\varepsilon_{max} = 60.13\ \mu\varepsilon$$

So we can get that $K_p = 1.314$.

The LDC of two kinds of TRGR derives from experimental study is consistent with theoretical calculation result . This indicates the theoretical calculation method by applying GEM is feasible.

5 Conclusions

1) A theoretical calculation method of LDC in TRGR by applying FEM and GEM is presented in this paper. Thus provides theoretically satisfying design foundation for load-equilibrating mechanism in TRGR.

2) The LDC of two kinds of TRGR is calculated not only in theory but also from experiment. On condition of the same manufacturing and assembling errors, the second kind of TRGR, whose eccentric phase difference equals 180°, is dynamically balanced on inertial force and its LDC is smaller than that of the first kind of TRGR, whose eccentric phase difference equals 120°. In other words, its load distribution capacity is superior to that of the first kind of TRGR.

3) The research indicates that the LDC of two kinds of TRGR derives from experimental study is consistent with the theoretical calculation result. That is to say, the theoretical calculation method by applying GEM is feasible.

References:

[1] CHEN Zongyuan, LIU Zhaowen, WANG Zhide, *et al.* Three-Ring Reducing Speed (Increasing Speed) Transmission Device [P]. P. R. China: CN 85 1 06692A, 1987-01-05 (in Chinese).

[2] LI Huamin, LIANG Yongsheng, XIN Shaojie. Dynamic Balance, Load Equilibrating and Vibration Reducing Two-stage Three Ring Gear Reducer [P]. P. R. China: ZL 00 2 34570.6, 2002-03-06 (in Chinese).

[3] ZHANG Shaoming. Analysis and calculation of floating magnitude in planetary transmission [J]. Automobile and Highway, 1978 (1): 780-782 (in Chinese).

[4] SDRC Co Ltd. I-DEAS Manual [M]. [s. l.]: [s. n.], 1995.

[5] LI Huamin, LIANG Yongsheng, XIN Shaojie. Study on load equilibrating and vibration reducing of three ring gear reducer [A]. 8th International ASME Power Transmission and Gearing Conference [C]. DETC 2000/PTG-14456: 1-5.

[6] WANG Shitong, YAN Huanxin, LIU Rongqiang, *et al.* Experimental study on Load equilibration and vibration reduction in 3-ring-gear reducer [J]. Journal of Harbin Institute of Technology, 1997, E-4(3): 43-47.